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# Thermometry by parallel arrays of ultrasmall double tunnel junctions

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**Abstract.** We propose a single-electron tunnelling thermometer, based on a parallel array of double tunnel junctions, which covers the temperature range from about 30 mK to about 4 K. The calibration curve has been derived analytically, and investigated numerically including co-tunnelling. The influence of background charge, which is a material problem, has also been studied.

Operative low-temperature thermometry methods, based on various physical principles, to a large extent cover the experimentally relevant temperature regime down to the mK range [1]. The $^{60}$Co thermometer operates at temperatures of about 30 mK, whereas various types of thermometers are operable above liquid He temperature. In the temperature range from about 30 mK to liquid He temperature, however, there is at present a lack of suitable thermometers. In this paper, we present a new thermometry method which is especially suited for the experimentally relevant temperature range between 30 mK and 4 K. The method is based on the temperature-dependent tunnelling probability of an ultrasmall double tunnel junction.

In ultrasmall tunnel junctions the Coulomb interaction produces the single-electron tunnelling (SET) phenomenon. The temperature characteristics of SET in tunnel junctions allows the fabrication of a new type of thermometer [2–4]. The simplest SET thermometer is a symmetric double tunnel junction (SDTJ) [3, 4] (shown schematically in the inset of figure 1), where each junction is characterized by a junction capacitance $C$ and a junction resistance $R$. It has been shown [2–4] that the normalized conductance $2RG(T)$ of a SDTJ at zero bias voltage is a universal function of the normalized temperature $k_B T/E_C$, where $E_C = e^2/2C_\Sigma$ is the Coulomb charging energy with the effective capacitance $C_\Sigma = 2C$. This universal function is plotted in figure 1, and can be approximated [4] by

$$G(T) = [1 - \tanh(0.3E_C/k_B T)]/(2R). \qquad (1)$$

Because of its universality, $G(T)$ serves as a calibration curve to determine the temperature by measuring the conductance of a SDTJ at zero bias voltage.

It is well known that besides the SET, higher-order co-tunnelling [5] processes also contribute to the conductance. For the devices investigated in this paper, we need only to

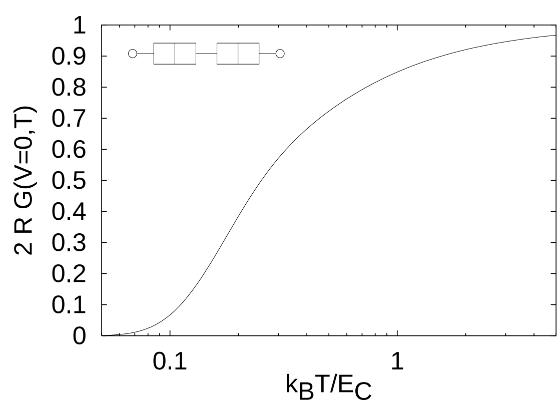


**Figure 1.** The universal functional dependence of the normalized conductance $2RG(T)$ on the normalized temperature $k_B T/E_C$ of a SDTJ (shown in the inset) at zero bias voltage $V = 0$.

consider incoherent co-tunnelling. Its contribution to the conductance at zero bias voltage is given by [5, 6]

$$G_{in}(T) = \begin{cases} \dfrac{4\pi\hbar}{3e^2R^2}\left(\dfrac{k_B T}{E_C}\right)^2 & \text{if } k_B T \ll E_C \\ \dfrac{\hbar a k_B T}{4e^2R^2}\operatorname{sech}^2\dfrac{E_C}{2k_B T} & \text{if } k_B T \lesssim E_C. \end{cases} \qquad (2)$$

$a$ is a constant associated with the electron–electron interaction in the metallic island of the SDTJ, and is connected to the electron–electron scattering time $\tau$ by [7] $\hbar/\tau = a(k_B T)^2$. To calculate the contribution of co-tunnelling, we will use the value $a = 1.82$ eV$^{-1}$ of Al, obtained from Landau–Fermi liquid theory [8] in terms of the s–p approximation of the angular average [9]. Due to lack of experimental data for Al we compared the results of experiment [10] and theory for Cu and found reasonable agreement (experiment: $a \approx 2.77$ eV$^{-1}$, theory:

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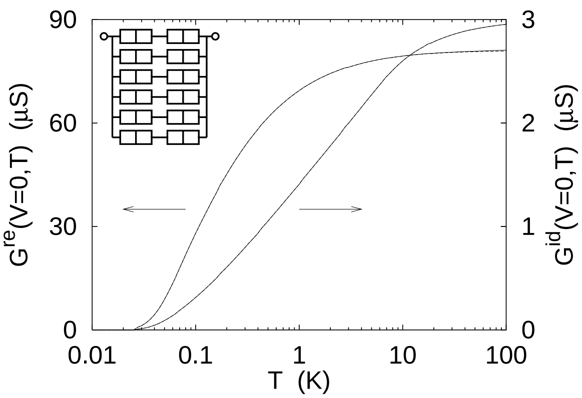


**Figure 2.** The zero bias conductance of a parallel array of 6 SDTJs (shown in the inset with sample structure given in the text) as a function of temperature for the *ideal* limit ($G^{\rm id}(T)$) and the *realistic* limit ($G^{\rm re}(T)$).

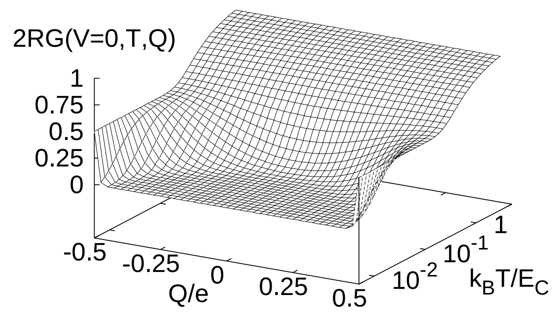


**Figure 3.** Effect of island offset charge $Q$ on the universal functional dependence of $2RG(T)$ on $k_{\rm B}T/E_{\rm C}$ at zero bias voltage. The behaviour is periodic in $Q$ with period $e$.

$a \approx 3.02$ eV$^{-1}$). We should mention that the above relation between $a$ and $\tau$ was derived [7] under the condition $\hbar/\tau \ll E_{\rm C}$, which is valid for the range of $E_{\rm C}$ and the temperature range considered in this article.

To calibrate a SET thermometer one should use the total conductance $G_{\rm t}(T) \equiv G(T) + G_{\rm in}(T)$. However, the co-tunnelling can be effectively suppressed in a series array of tunnelling junctions and, therefore, a very accurate SET thermometer has been fabricated with a series array of 40 identical junctions [2]. Then its temperature calibration has the features of a single SDTJ given in equation (1). The operation temperature range $T_- < T < T_+$ of such a SET thermometer centres around $E_{\rm C}/k_{\rm B}$, since in this situation the significant electron (or hole) occupation probability in the vicinity of the energy $E_{\rm F} + E_{\rm C}$ (or $E_{\rm F} - E_{\rm C}$) is strongly temperature dependent, where $E_{\rm F}$ is the Fermi energy in the contact reservoirs. The precise values of $T_-$ and $T_+$ are difficult to predict because of the external influences such as noise and sensitivity of the electronic circuit used. Nevertheless, when all relevant factors are taken into account, the device has a minimal resolution $\mathcal{G}$ of $\partial G_{\rm t}(T)/\partial T$. A reasonable estimate [4] gives $T_+/T_- \simeq 10$. That is, the presently available SET thermometer works in a temperature range about one order of magnitude around $E_{\rm C}/k_{\rm B}$. Consequently, a single SET thermometer, using a series array of many identical tunnel junctions, cannot cover the desired temperature from about 30 mK to about 4 K.

In this paper we propose a new type of SET thermometer, using a parallel array of selective SDTJs, which can cover the entire desired range of temperatures. Let us consider a parallel array of $N$ SDTJs (labelled by $i = 0, 1, 2, \ldots, N-1$) as shown in the inset in figure 2. For the $i$th SDTJ, the junction resistance is $R_i$ and the junction capacitance is $C_i = C_{\Sigma i}/2$. The cross-capacitances between different SDTJs can be effectively reduced by using a special fabrication method to spatially separate the metallic islands and/or to shield the junctions in different SDTJs. Recent experiments [11] show that even in the case of close SDTJs ($\simeq 100$ nm) cross-talk between them is small. Therefore, we neglect the effect of cross-capacitances in our theoretical analysis. In this case the SET process in one SDTJ is decoupled from the SET processes in other SDTJs. In other words, the conductance of each SDTJ in the parallel array can be calculated independently. We will order the $N$ SDTJs such that $C_i < C_j$ if $i > j$, and so the corresponding Coulomb charging energies are also ordered as $E_{{\rm C}_i} = e^2/(2C_{\Sigma_i}) > e^2/(2C_{\Sigma_j}) = E_{{\rm C}_j}$ if $i > j$. As the temperature increases, the $N$ SDTJs become conductive one after the other following the increasing order of $i$ from $i = 0$ to $i = N-1$. In this *cascade*, each of the $N$ SDTJs contributes a partial conductance independently, and the total conductance $G_{\rm PA}(T)$ of the parallel array at zero bias voltage is just the sum of these partial conductances. By a proper choice of the set of device parameters $\{C_i, R_i\}$, the normalized $G_{\rm PA}(T)$ can be a smooth universal function of the normalized temperature, which will serve as the calibration curve of the parallel array SET thermometer.

The structure of the parallel array SET thermometer we propose in this paper has the capacitances specified as $C_i = x^{-i}C_0$ with $x \geq 1$. When a SDTJ is fabricated, the relation between $R_i$ and $C_i$ depends on certain specific details [12], which are outside the scope of this work. Given such a relation, the conductance $G_{\rm PA}(T)$ can be obtained numerically as a function of the temperature. However, it will be valuable to derive an analytical expression of $G_{\rm PA}(T)$. For this purpose, here we assume a general relation $R_i = y^i R_0$ with $1 \leq y \leq x$. The case $y = 1$ (independence of the junction resistances on the capacitances) is called the *ideal* limit. On the other hand, if $R_iC_i$ = constant then $y = x$, and we call this case the *realistic* limit. For general values of $x$, $y$ and $N$, the analytical form of $G_{\rm PA}(T)$ is too complicated to be presented here. In the limit $N \to \infty$, however, $G_{\rm PA}(T)$ can be cast into a comprehensible form,

$$G_{\rm PA}(T) = \frac{1}{(y-1)R_0}[y - (2E_{{\rm C}_0}/k_{\rm B}T)^{(\ln y)/\ln x}]. \qquad (3)$$

The smallest metallic islands which can be fabricated with presently available technology [13] have capacitances as low as $C_{\Sigma,\min} \simeq 10^{-16}$–$10^{-17}$ F, which corresponds to a temperature of about 10 K. Therefore, if we are interested in a parallel array SET thermometer operating below 4 K, all those SDTJs in the parallel array with $C_{\Sigma,i} < C_{\Sigma,\min}$, and so with corresponding operating temperature higher than 10 K, will not contribute to $G_{\rm PA}(T)$. In this case, $G_{\rm PA}(T)$ is indiscernible from the asymptotic result for $N \to \infty$ of (3). The result of (3) does not include higher-order co-tunnelling contributions. We have checked both analytically and numerically that for a reasonable number of SDTJs in the parallel array, the contribution of co-tunnelling to $G_{\rm PA}(T)$ is negligible.

From the calibration curve (3) we deduce that the lower bound of operation temperature is $T_- < T_0 \equiv E_0/k_{\rm B}$. The upper bound $T_+$ is determined, as we mentioned above, by the condition $[\partial G_{\rm PA}(T)/\partial T]_{T=T_+} = \mathcal{G}$, where $\mathcal{G}$ is the device's resolution limit of $\partial G_{\rm PA}(T)/\partial T$. From (3) we obtain

$$T_+ = \frac{E_0}{k_{\rm B}} \left[ \frac{1}{\mathcal{G}} \, \frac{k_{\rm B}}{E_0 R_0 (y-1)} \, \frac{\ln y}{\ln x} \right]^{(\ln x)/\ln(xy)}. \qquad (4)$$

This reduces to

$$T_+^{\rm id} = \frac{1}{\mathcal{G} R_0 \ln x} \qquad (5)$$

for the ideal limit, and to

$$T_+^{\rm re} = [T_0 T_+^{\rm id} (\ln x)/(x-1)]^{1/2} \qquad (6)$$

for the realistic limit. Note that large parallel arrays allow $x \to 1$ which in turn yields $T_+^{\rm re} \to (T_0 T_+^{\rm id})^{1/2}$, which is the geometric mean of the ideal limit and the lower bound.

We have set $C_0 = 5 \times 10^{-15}$ F, $R_0 = 10^6\ \Omega$ and $x = 2.5$ to calculate the conductance of a parallel array of $N = 6$ SDTJs at zero bias voltage, including co-tunnelling. The results are plotted in figure 2 for the ideal limit (curve $G^{\rm id}(T)$) and for the realistic limit (curve $G^{\rm re}(T)$). These curves cannot be distinguished from the corresponding curves without the contribution of co-tunnelling.

One problem of the presented SET thermometer is the so-called *background charges* trapped within the junction barriers or the substrate [11, 14]. When these charges move, the energy of the island changes and alters the current–voltage characteristic. Although this material problem is outside the scope of this work, the effect of background charges can be studied implicitly via the effect of an island offset charge $Q$ on the conductance. Let us consider the simple case of a SDTJ. Using the orthodox theory [15, 16], we have calculated the normalized conductance $2RG(T, Q)$ as a function of the normalized temperature $k_{\rm B}T/E_{\rm C}$ and the normalized island offset charge $Q/e$. The result is plotted in figure 3. We see that if $|Q|/e \ll 0.5$, the functional dependence of $2RG(0, T, Q)$ on $k_{\rm B}T/E_{\rm C}$ is insensitive to the variation of $Q/e$. Hence, a reduction of the charge fluctuations to values below $e$ would circumvent the problem with regard to the proposed application. This is likely to be achieved with new substrate materials. Since charge fluctuations enlarge the conductance of the SDTJ (cf figure 3), their time scale might allow one to use the minimal conductance value to determine the temperature.

To conclude, we have presented a novel and experimentally accessible method for low-temperature thermometry, based on the temperature-dependent conductance of a parallel array of symmetric double tunnel junctions. Using realistic parameter values for the capacitances and resistances of the tunnel junctions, we find that it is operable in the relevant temperature regime of several ten mK to liquid He temperature.

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